

## Quartic anomalous couplings at LEP

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### Abstract

The search for quartic anomalous gauge couplings at LEP requires appropriate predictions for the radiative processes  $e^+e^- \rightarrow \nu\bar{\nu}\gamma\gamma$ ,  $e^+e^- \rightarrow q\bar{q}\gamma\gamma$  and  $e^+e^- \rightarrow 4 \text{ fermions} + \gamma$ . Matrix elements are exactly computed at the tree level, and the effects of anomalous couplings and initial-state radiation are included. Comparisons with results and approximations existing in the literature are shown and commented. Improved versions of the event generators **NUNUGPV** and **WRAP** are made available for experimental analysis.

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*Keywords:* electron-positron collision, radiative events, anomalous couplings, Monte Carlo.

# 1 Introduction

Despite of the striking success of the Standard Model (SM) in accommodating the precision data collected at high-energy colliders, important tests of the theory, such as the non-abelian nature of the gauge symmetry and the mechanism of electroweak symmetry breaking, are still at a beginning stage. To this end, gauge-boson self interactions play a key role. Presently, triple gauge couplings are being probed at LEP [1] and Tevatron [2], while only very recently direct measurements of quartic couplings became available through the study of radiative events at LEP [3]-[6]. Actually, events with one or two isolated, hard photons are analyzed at LEP to search for anomalies in the sector of quartic gauge-boson couplings. Only vertices involving at least one photon can be constrained since quadrilinear interactions containing four massive gauge bosons give rise to a three massive gauge boson final state and are therefore beyond the potentials of LEP due to lack of phase space. The processes considered in the experimental analyses are  $e^+e^- \rightarrow W^+W^-\gamma$ ,  $e^+e^- \rightarrow Z\gamma\gamma$  and  $e^+e^- \rightarrow \nu\bar{\nu}\gamma\gamma$  [3]-[6]. The  $W^+W^-\gamma$  signature, which yields a four fermion plus gamma final state, is interesting in order to test  $WW\gamma\gamma$  and  $WWZ\gamma$  vertices. For the  $Z\gamma\gamma$  events, the final states due to the hadronic decays of the  $Z$  boson with two jets and two visible photons are selected to probe the purely anomalous vertex  $ZZ\gamma\gamma$ , which is of particular interest being absent in the SM at tree level. The final state with two neutrinos and two acoplanar photons allows to study the quartic  $WW\gamma\gamma$  and  $ZZ\gamma\gamma$  interactions.

For tests of the SM and searches for new physics beyond it, anomalous quartic gauge-boson couplings are important because, as widely discussed in the literature [7], they offer a window on the mechanism of symmetry breaking and it is possible to imagine extensions of the SM that alter the quartic vertices without modifying the trilinear interactions. For this reason, phenomenological analyses of anomalous quartic interactions have already appeared in the literature, by considering the potentials of  $\gamma\gamma$  [8],  $e\gamma$  [9] and  $e^+e^-$  [10]-[13] colliders and very recently also of hadron machines [14]. The tightest constraints on quartic anomalous couplings come from electroweak precision data, as discussed in ref. [14]. However, present measurements at LEP2 can investigate photonic quartic couplings for the first time in a direct way through the study of the final states above discussed. In the light of these experimental analyses, the aim of this letter is to present exact SM calculations of the processes  $e^+e^- \rightarrow 4 \text{ fermions } (4f) + \gamma$ ,  $e^+e^- \rightarrow q\bar{q}\gamma\gamma$  and  $e^+e^- \rightarrow \nu\bar{\nu}\gamma\gamma$ , including the effects of quartic anomalous gauge couplings (QAGC) and of the most important radiative corrections due to initial-state radiation (ISR). The sensitivity of LEP searches is studied in comparison with approximate results existing in the literature. It turns out that the  $Z\gamma\gamma$  approximation works well for the  $q\bar{q}\gamma\gamma$  and  $\nu\bar{\nu}\gamma\gamma$  channels when appropriate cuts around the  $Z$  mass are imposed, while the  $WW\gamma$  approximation can significantly differ from the exact  $4f + \gamma$  calculation, even in the presence of invariant mass cuts around the  $W$  mass. It is also shown that the  $\nu\bar{\nu}\gamma\gamma$  final state with appropriate cuts on the recoil mass can be successfully exploited to extract limits on neutral QAGC, as a complementary channel to the  $q\bar{q}\gamma\gamma$  process.

## 2 Theoretical approach

The theoretical framework of interest is the formalism of electroweak chiral lagrangians. In such a scenario, QAGC involving four massive gauge bosons emerge as operators of dimension four at next-to-leading order, while QAGC with at least one photon originate from six (or higher) dimensional operators at next-to-next-to-leading order. They are said genuinely anomalous if they do not induce new trilinear gauge interactions.

Anomalous  $WW\gamma\gamma$  and  $ZZ\gamma\gamma$  vertices were originally introduced in ref [8]. In this paper, the authors show that, by assuming  $C$  and  $P$  conservation and further imposing  $U(1)_{em}$  gauge invariance and  $SU(2)_c$  custodial symmetry, two independent Lorentz structures contribute to  $WW\gamma\gamma$  and  $ZZ\gamma\gamma$  interactions according to the following lagrangians

$$\begin{aligned}\mathcal{L}_0 &= -\frac{e^2}{16} \frac{a_0}{\Lambda^2} F_{\mu\nu} F^{\mu\nu} \vec{W}^\alpha \cdot \vec{W}_\alpha \\ \mathcal{L}_c &= -\frac{e^2}{16} \frac{a_c}{\Lambda^2} F_{\mu\alpha} F^{\mu\beta} \vec{W}^\alpha \cdot \vec{W}_\beta,\end{aligned}\tag{1}$$

where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the electromagnetic field tensor, and  $\vec{W}$  is a  $SU(2)$  triplet describing the  $W$  and  $Z$  physical fields, i.e.

$$\vec{W} = \begin{pmatrix} \frac{1}{\sqrt{2}}(W_\mu^+ + W_\mu^-) \\ \frac{i}{\sqrt{2}}(W_\mu^+ - W_\mu^-) \\ Z_\mu / \cos \theta_w \end{pmatrix},$$

$\cos \theta_w$  being the cosine of the weak mixing angle. In eq. (1)  $a_0$  and  $a_c$  are (dimensionless) anomalous couplings, divided by an energy scale  $\Lambda$ , which has the meaning of scale of new physics. Generally speaking,  $\Lambda$  is in principle unknown and model-dependent. However, the ratios  $a_i/\Lambda^2$  entering the phenomenological lagrangians can be meaningfully extracted from the data in a model-independent way.

An anomalous  $WWZ\gamma$  interaction has also been advocated in refs. [9]-[11]. The corresponding phenomenological lagrangian reads as follows

$$\mathcal{L}_n = -\frac{e^2}{16} \frac{a_n}{\Lambda^2} \epsilon_{ijk} W_{\mu\alpha}^i W_\nu^j W^{k\alpha} F^{\mu\nu},\tag{2}$$

where  $\vec{W}_{\mu\nu}$  is the  $SU(2)$  field strength tensor, and  $a_n/\Lambda^2$  the anomalous coupling. Although  $U(1)_{em}$  and  $SU(2)_c$  conserving, the lagrangian of eq. (2) violates  $C$  and  $CP$ , as noticed in the literature [13]. The anomalous couplings  $a_0, a_c, a_n$  entering eqs. (1)-(2) are presently constrained by LEP collaborations [3]-[6]. Under the assumptions of local  $U(1)_{em}$  invariance and global custodial  $SU(2)_c$  symmetry, two additional operator structures which violate  $P$  have been very recently proposed in ref. [15].

A complete and general analysis of photonic quartic couplings has been performed in ref. [13]. In this paper it is shown that, by imposing  $C$ ,  $P$  and  $U(1)_{em}$  invariance, nine independent Lorentz structures do contribute to  $WW\gamma\gamma$ ,  $ZZ\gamma\gamma$  and  $WWZ\gamma$  vertices. It is further demonstrated by the authors that, by embedding these structures in  $SU(2) \times$

$U(1)$  gauge invariant and  $SU(2)_c$  symmetric combinations, fourteen  $C$  and  $P$  conserving operators are allowed, with  $k_j^i$  parameters, which parameterize the strength of anomalous couplings.

Following ref. [13], the nine independent operator structures contributing to the vertices analyzed at LEP have been implemented directly at the lagrangian level in the **ALPHA** algorithm [16], according to the formula

$$L_{QAGC} = W_1 + W_2 + Z_1 + Z_2 + W_0^Z + W_c^Z + W_1^Z + W_2^Z + W_3^Z, \quad (3)$$

where the Lorentz structure of the operators is given by:

$$\begin{aligned} W_1 &= a_{w1} F_{\mu\nu} F^{\mu\nu} W_\rho^+ W^{-\rho} \\ W_2 &= a_{w2} F_{\mu\nu} F^{\mu\rho} W^{+\nu} W_\rho^- + \text{h.c.} \\ Z_1 &= a_{z1} F_{\mu\nu} F^{\mu\nu} Z_\rho Z^\rho \\ Z_2 &= a_{z2} F_{\mu\nu} F^{\mu\rho} Z^\nu Z_\rho \\ W_0^Z &= a_{wz0} F_{\mu\nu} Z^{\mu\nu} W_\rho^+ W^{-\rho} \\ W_c^Z &= a_{wzc} F_{\mu\nu} Z^{\mu\rho} W^{+\nu} W_\rho^- + \text{h.c.} \\ W_1^Z &= a_{wz1} F_{\mu\nu} W^{+\mu\nu} Z^\rho W_\rho^- + \text{h.c.} \\ W_2^Z &= a_{wz2} F_{\mu\nu} W^{+\mu\rho} Z^\nu W_\rho^- + \text{h.c.} \\ W_3^Z &= a_{wz3} F_{\mu\nu} W^{+\mu\rho} Z_\rho W^{-\nu} + \text{h.c.}, \end{aligned} \quad (4)$$

the  $a_i$  being coefficients of dimension  $M^{-2}$ .

The first two structures refer to  $WW\gamma\gamma$  interactions, the third and fourth ones to  $ZZ\gamma\gamma$  interactions, while the remaining five affect the  $WWZ\gamma$  vertex. It is worth noticing that, by virtue of eq. (4), both parameterizations available in the literature for QAGC, namely the parameterization in terms of  $a_0, a_c, a_n$  couplings and the one in terms of  $k_j^i$  coefficients, can be obtained by means of appropriate relations between the  $a_i$  parameters. For example, assigned  $a_0, a_c$  and  $a_n$ , the following relations for the  $a_i$  coefficients of eq. (4) hold

$$\begin{aligned} a_{w1} &= -\frac{e^2}{8\Lambda^2} a_0 \\ a_{z1} &= -\frac{e^2}{16 \cos^2 \theta_w \Lambda^2} a_0 \\ a_{w2} &= -\frac{e^2}{8\Lambda^2} a_c \\ a_{z2} &= -\frac{e^2}{16 \cos^2 \theta_w \Lambda^2} a_c \\ a_{wzc} &= i \frac{e^2}{16 \cos \theta_w \Lambda^2} a_n \\ a_{wz2} &= i \frac{e^2}{16 \cos \theta_w \Lambda^2} a_n \\ a_{wz3} &= -i \frac{e^2}{16 \cos \theta_w \Lambda^2} a_n. \end{aligned} \quad (5)$$

The implementation in **ALPHA** has been carefully cross-checked by an analytical calculation of the scattering processes  $WW \rightarrow \gamma\gamma$ ,  $ZZ \rightarrow \gamma\gamma$  and  $WW \rightarrow Z\gamma$  (and its permutations), finding perfect agreement.

In order to provide valuable tools for experimental data analysis, the version of **ALPHA** including the lagrangian of eq. (4) has been interfaced to the Monte Carlo generators **NUNUGPV** [17] and **WRAP** [18], for a complete study of QAGC in radiative events at LEP.

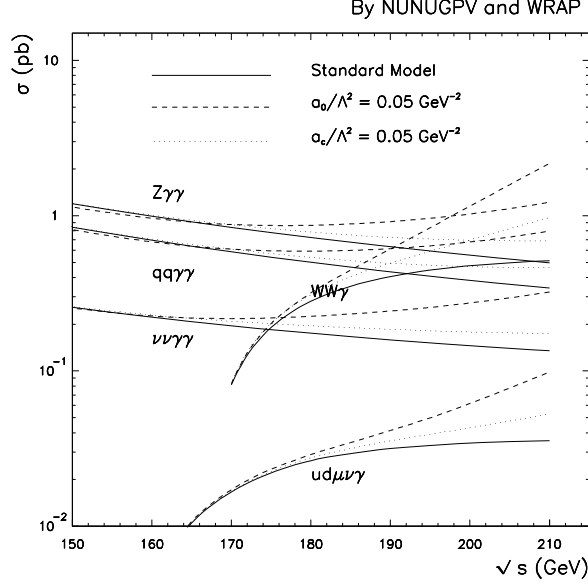


Figure 1: The cross sections in the absence (solid line) and in the presence (dashed and dotted lines) of anomalous parameters  $a_0$  and  $a_c$  ( $a_0/\Lambda^2 = a_c/\Lambda^2 = 0.05 \text{ GeV}^{-2}$ ) for the processes  $e^+e^- \rightarrow Z\gamma\gamma, WW\gamma, q\bar{q}\gamma\gamma, \nu\bar{\nu}\gamma\gamma, u\bar{d}\mu\bar{\nu}_\mu\gamma$ , as functions of the c.m. energy in the LEP2 energy range.

The  $\nu\bar{\nu}\gamma\gamma$  final state can be simulated by means of NUNUGPV, which includes exact SM matrix elements and  $p_t$ -dependent QED Structure Functions (SF) to account for ISR. The other processes of interest at LEP can be studied by using WRAP, which is based on the exact matrix elements for the (charged-current) reactions  $e^+e^- \rightarrow 4f+\gamma$  and the process  $e^+e^- \rightarrow q\bar{q}\gamma\gamma$  and the same treatment as NUNUGPV for ISR. By means of WRAP, predictions for the inclusive final states  $WW\gamma$  and  $Z\gamma\gamma$  can be also obtained, especially in order to compare with results existing in the literature treating the  $W$  and  $Z$  bosons in the on-shell approximation, as those of refs. [11, 13].

### 3 Numerical results and discussion

In order to test the implementation of QAGC, comparisons with the available results have been performed. A first comparison has been done with the plots published in ref. [13] for the dependence of the  $WW\gamma$  and  $Z\gamma\gamma$  cross sections on the anomalous parameters  $k_i^j$  at  $\sqrt{s} = 200 \text{ GeV}$ , using the same set of input parameters and cuts. The results of such a comparison between the results of ref. [13] and the predictions of the Monte Carlo WRAP show a very satisfactory agreement, testifying a correct implementation of  $WW\gamma\gamma$ ,

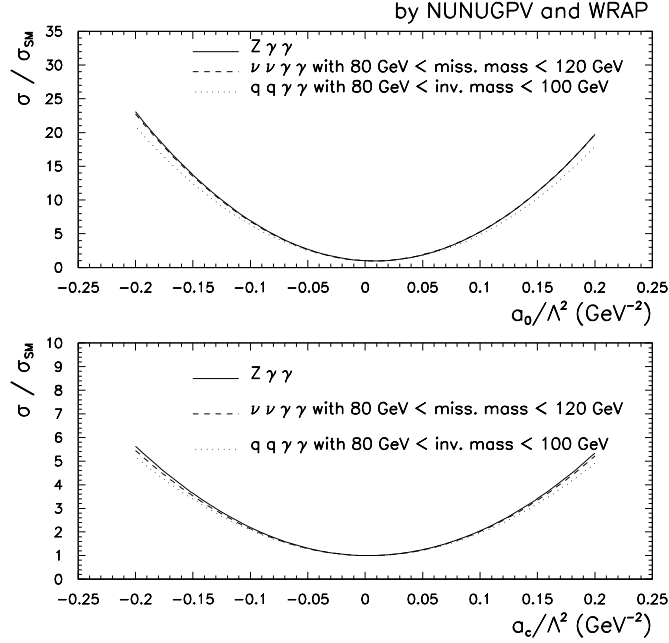


Figure 2: The sensitivity of the  $q\bar{q}\gamma\gamma$  and  $\nu\bar{\nu}\gamma\gamma$  final states to  $a_0$  and  $a_c$  anomalous couplings in comparison with the  $Z\gamma\gamma$  approximation, at  $\sqrt{s} = 200 \text{ GeV}$ .

$ZZ\gamma\gamma$  and  $WWZ\gamma$  couplings in the present calculation.

As far as the parameterization in terms of  $a_0$  and  $a_c$  anomalous couplings is concerned, two further comparisons have been performed. The first one has been done with the results of ref. [13] for the dependence of the  $WW\gamma$  cross section on  $a_0$ , finding perfect agreement. The second one has been performed with the results published in ref. [11] for the dependence of the  $WW\gamma$  and  $Z\gamma\gamma$  cross sections on  $a_0$  and  $a_c$  parameters, using the same set of cuts. The predictions obtained with WRAP differ from those of ref. [11]. More precisely, an opposite sign is present for the relative effects of  $a_0$  and  $a_c$  on the  $WW\gamma$  cross section, as noticed in ref. [15] and confirmed [19] by the authors of ref. [11], whereas this is not the case for the  $Z\gamma\gamma$  final state. Work is in progress in order to clarify this point [19].

For the results shown in the following, the input parameters used are:

$$\begin{aligned}
 G_F &= 1.16637 \cdot 10^{-5} \text{ GeV}^{-2} & M_Z &= 91.1867 \text{ GeV} \\
 M_W &= 80.35 \text{ GeV} & \sin^2 \theta_w &= 1 - M_W^2 / M_Z^2 \\
 \Gamma_Z &= 2.49471 \text{ GeV} & \Gamma_W &= 2.04277 \text{ GeV}
 \end{aligned} \tag{6}$$

The cuts adopted are:  $E_\gamma \geq 5 \text{ GeV}$ ,  $15^\circ \leq \vartheta_\gamma \leq 165^\circ$  as detection criteria for the

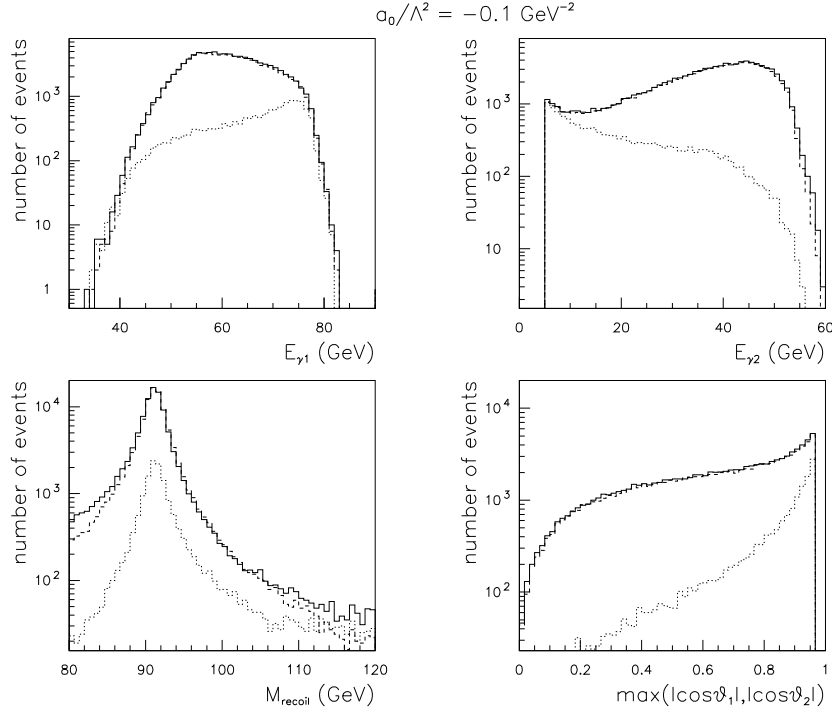


Figure 3: Distribution of the energy of the most energetic photon ( $E_{\gamma 1}$ ), the energy of the second most energetic photon ( $E_{\gamma 2}$ ), the recoil mass ( $M_{\text{recoil}}$ ) and the polar angle of the most forward photon ( $\max(|\cos \vartheta_1|, |\cos \vartheta_2|)$ ), for the  $\nu\bar{\nu}\gamma\gamma$  final state, with  $a_0/\Lambda^2 = -0.1 \text{ GeV}^{-2}$  and  $\sqrt{s} = 200 \text{ GeV}$ . Dotted histograms: SM calculation; solid histograms: full  $\nu\bar{\nu}\gamma\gamma$  calculation for  $a_0/\Lambda^2 = -0.1 \text{ GeV}^{-2}$ ; dashed histograms:  $\nu_\mu\bar{\nu}_\mu\gamma\gamma$  prediction for  $a_0/\Lambda^2 = -0.1 \text{ GeV}^{-2}$ .

observed photons, together with a photon-final state charged fermion separation cut of  $5^\circ$ . Invariant mass cuts of the kind  $80 \text{ GeV} \leq M_{q\bar{q}} \leq 100 \text{ GeV}$  for the  $e^+e^- \rightarrow q\bar{q}\gamma\gamma$  process and  $M_{u\bar{d}} \geq 10 \text{ GeV}$  for  $e^+e^- \rightarrow u\bar{d}\mu\bar{\nu}_\mu\gamma$  process are also imposed.

Figure 1 shows the total cross sections, as obtained by means of the event generators NUNUGPV and WRAP, of all the radiative processes accessible to the LEP investigation, as functions of the center of mass (c.m.) energy in the LEP2 energy range. The solid lines correspond to the pure SM predictions; the dashed and dotted lines illustrate the effects of anomalous couplings  $a_0/\Lambda^2 = 0.05 \text{ GeV}^{-2}$  and  $a_c/\Lambda^2 = 0.05 \text{ GeV}^{-2}$ , respectively. The unitarity violations induced by the anomalous couplings are clearly visible, their effects growing as the c.m. energy increases.

In Fig. 2 the sensitivity of the final states  $q\bar{q}\gamma\gamma$  (dotted line) and  $\nu\bar{\nu}\gamma\gamma$  (dashed line), as obtained by means of the full calculation, to the  $a_0$  and  $a_c$  parameters is shown at the c.m. energy of 200 GeV, in comparison with the  $Z\gamma\gamma$  approximation (solid line). The predictions for the  $q\bar{q}\gamma\gamma$  channel refer to a cut on the invariant mass of the jet-jet system

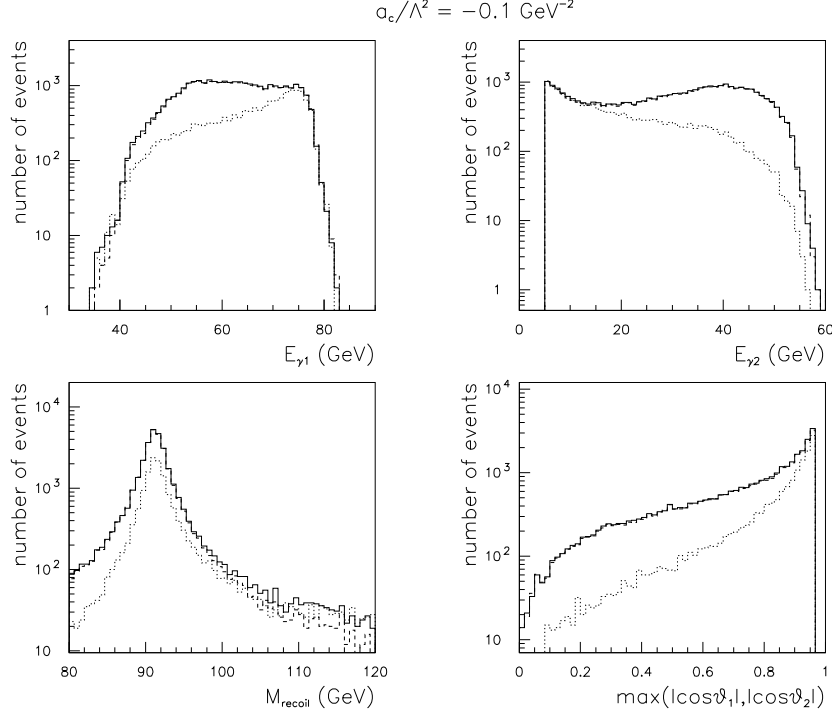


Figure 4: The same as Fig. 3, for  $a_c/\Lambda^2 = -0.1 \text{ GeV}^{-2}$ .

around the  $Z$  mass ( $80 \text{ GeV} \leq M_{q\bar{q}} \leq 100 \text{ GeV}$ ), while for the  $\nu\bar{\nu}\gamma\gamma$  final state a cut on the recoil mass again around the  $Z$  mass ( $80 \text{ GeV} \leq M_{recoil} \leq 120 \text{ GeV}$ ) has been imposed. These cuts, also typically adopted in the experimental analysis, are required in order to compare with the  $Z\gamma\gamma$  approximation, which, according to the results available in the literature [11, 13], assumes the  $Z$  boson as an on-shell particle. Therefore, this comparison allows to quantify the effects due to the  $\gamma$ - $Z$  interference in the  $q\bar{q}\gamma\gamma$  channel and to  $W$ - $Z$  interference in the  $\nu\bar{\nu}\gamma\gamma$  one, as well as the effects of the off-shellness of the  $Z$  boson, in the extraction of limits on the QAGC. It can be seen from Fig. 2 that the  $Z\gamma\gamma$  calculation is a very good approximation of the full prediction, which includes interference contributions. A detailed numerical investigation shows an agreement between the integrated cross sections at the per cent level as a function of  $a_0$  and  $a_c$  variations inside the presently allowed experimental constraints, thus illustrating the reliability of the  $Z\gamma\gamma$  approximation in view of the expected experimental precision.

This conclusion also holds for the differential distributions mostly sensitive to QAGC and considered in the experimental studies, as can be noticed from Figs. 3-4, showing the energy of the most energetic photon ( $E_{\gamma_1}$ ), the energy of the second most energetic photon ( $E_{\gamma_2}$ ), the recoil mass ( $M_{recoil}$ ) and the polar angle of the most forward photon ( $\max(|\cos\vartheta_1|, |\cos\vartheta_2|)$ ), for  $a_0/\Lambda^2 = -0.1 \text{ GeV}^{-2}$  (Fig. 3) and  $a_c/\Lambda^2 = -0.1 \text{ GeV}^{-2}$



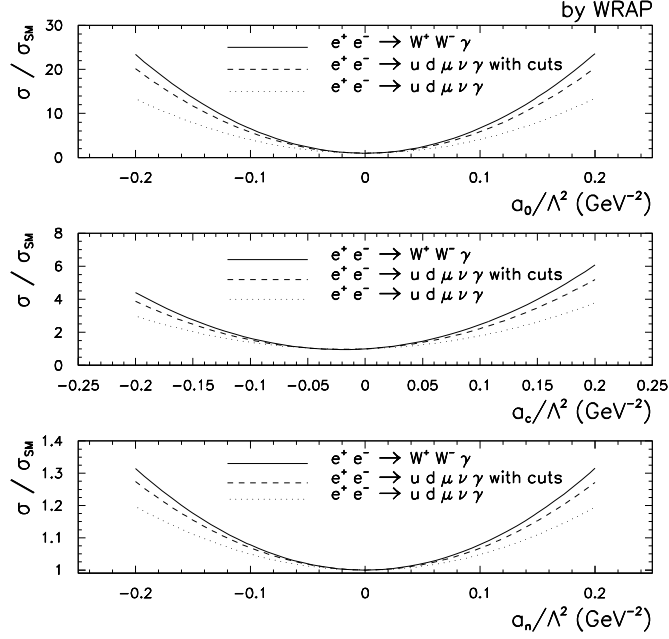


Figure 5: The sensitivity of the  $u\bar{d}\mu\bar{\nu}_\mu\gamma$  final state to  $a_0$ ,  $a_c$  and  $a_n$  anomalous couplings in comparison with the  $WW\gamma$  approximation, at  $\sqrt{s} = 200$  GeV.

(Fig. 4). In Figs. 3-4 the predictions obtained by means of the complete calculation for the  $\nu\bar{\nu}\gamma\gamma$  final state, including  $W$ - $Z$  interference, (solid histograms), are compared with the results relative to the  $\nu_\mu\bar{\nu}_\mu\gamma\gamma$  process, proceeding only via  $Z$ -boson exchange (dashed histograms). For the  $\nu\bar{\nu}\gamma\gamma$  final state, a cut on the recoil mass around the  $Z$  mass ( $80 \text{ GeV} \leq M_{\text{recoil}} \leq 120 \text{ GeV}$ ) has been imposed. For the sake of comparison, the SM predictions (dotted histograms) normalized to the same luminosity are also shown. The difference between the solid and dashed histograms indicates a very moderate contribution due to  $W$ - $Z$  interference, even for the large  $a_0$  and  $a_c$  values considered in the simulation.

A similar analysis is shown in Fig. 5 for a  $4f + \gamma$  final state, as computed by means of the exact calculation of WRAP, in comparison with the  $WW\gamma$  approximation considered in the literature [11, 13]. The sensitivity to the anomalous couplings  $a_0, a_c, a_n$  is shown for the  $4f + \gamma$  final state according to two different event selections: no cuts on the invariant masses of the decay products (dotted line) and cuts on the invariant masses of the decay products around the  $W$  mass, i.e.  $75 \text{ GeV} \leq M_{u\bar{d}, \mu\bar{\nu}_\mu} \leq 85 \text{ GeV}$ , (dashed line), in order to disentangle, as much as possible, the contributing Feynman graphs with two final-state resonant  $W$  bosons. The  $WW\gamma$  approximation (solid line) predicts a quite different sensitivity with respect to the complete  $4f + \gamma$  calculation.

By considering variations of the anomalous couplings within the allowed experimental bounds, differences at the ten per cent level are registered between the  $WW\gamma$  approximation and the  $4f + \gamma$  prediction, when invariant mass cuts are imposed in the  $4f + \gamma$  calculation. Notice that, if invariant mass cuts are not considered, the  $4f + \gamma$  cross section grows up by a factor of two with respect to the cross section in the presence of cuts. Therefore, the  $WW\gamma$  approximation should be employed with the due caution in QAGC studies, especially if one takes into account that exact  $4f + \gamma$  generators, such as WRAP [18] and RacoonWW [15], incorporate the effects of QAGC and are at disposal for such experimental studies.

For the sake of simplicity, only numerical results in the Born approximation have been presented in the paper. Anyway, as already remarked and previously discussed in detail in refs. [17, 18], the effects of ISR can be properly simulated by means of NUNUGPV and WRAP and should be considered in the experimental analysis since they tend to diminish the sensitivity on the QAGC at some per cent level, as proved by explicit numerical investigation.

## 4 Conclusions

The search for QAGC in radiative events at LEP demands precise predictions for the processes  $e^+e^- \rightarrow \nu\bar{\nu}\gamma\gamma$ ,  $e^+e^- \rightarrow q\bar{q}\gamma\gamma$  and  $e^+e^- \rightarrow 4 \text{ fermions} + \gamma$ . To this end, exact calculations of such processes have been presented, incorporating the contribution of QAGC and the large effect of ISR.

By means of the exact calculations, comparisons with approximate results existing in the literature have been performed. It turns out that, for the  $\nu\bar{\nu}\gamma\gamma$  and  $q\bar{q}\gamma\gamma$  final states, the  $Z\gamma\gamma$  approximation works well, being the interference effects present in the complete calculations confined at the per cent level, if appropriate cuts around the  $Z$ -boson mass are required. It has been also demonstrated that the  $\nu\bar{\nu}\gamma\gamma$  final state with appropriate cuts on the recoil mass can be successfully exploited to extract limits on neutral QAGC, as a complementary channel to the  $q\bar{q}\gamma\gamma$  one. As far as the  $4 \text{ fermions} + \gamma$  final states are concerned, significant differences are seen between the exact calculation and the  $WW\gamma$  approximation, even in the presence of invariant mass cuts around the  $W$ -boson mass.

In the spirit of the present study, a phenomenological analysis of anomalous gauge couplings in radiative events at the energies of future linear colliders, including the effects of beam polarization and beamsstrahlung, will be given elsewhere.

The exact calculations addressed in the present paper are available in the form of improved versions of the event generators NUNUGPV and WRAP <sup>1</sup>, which can be used for full experimental simulations.

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<sup>1</sup>The Monte Carlo programs NUNUGPV and WRAP can be downloaded from the WEB site <http://decux1.pv.infn.it/~nicrosi/programs.html>

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